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PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 58.

December 14th, 1846.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Rev. William Reeves, M. B., was elected a Member of the Academy.

The Rev. Charles Graves made some observations on the use of distributive signs of operation, both real and imaginary, in the construction of systems of algebra.

1. An algebra with two distributive signs, one real and the other imaginary, might be framed in the following manner:

We might define the sign $+$ as standing for a distributive operation, such that the repetition of it any integer number of times is still equivalent merely to $+$. We might suppose the nature of this operation known, and, on that account, call any quantity affected with the sign $+$ a *real* quantity. Along with this symbol we might employ a second, $+^{\frac{1}{2}}$, defining it as being the symbol of another distributive operation, such, that the twofold execution of it is equivalent to $+$. And the nature of this operation, denoted by $+^{\frac{1}{2}}$, might be supposed unknown, or in some sense transcendental as compared with $+$, so that it would be impossible to express a term affected with the sign $+^{\frac{1}{2}}$ by means of any combination of terms affected with the sign $+$. With this understanding we might call terms affected with $+^{\frac{1}{2}}$ *imaginary*, and the importance of the distinction between reals and imaginaries consists in this: that when we have an equation, $\mathrm{U} = 0$, involving quantities of both kinds, we may put the real and imaginary parts of U respectively equal to 0.

The following example will show the advantage of employing such a symbol. If we put $+^{\frac{1}{2}}\phi$ in place of $+\phi$ in the development of e^{ϕ}, we find

$$e^{+^{\frac{1}{2}}\phi} = \cos\phi +^{\frac{1}{2}} \sin\phi \qquad (1)$$

where $\cos\phi$ and $\sin\phi$ respectively stand for the series

$$1 + \frac{\phi^2}{1.2} + \frac{\phi^4}{1.2.3.4} + \&c.$$

$$\phi + \frac{\phi^3}{1.2.3} + \frac{\phi^5}{1.2.3.4.5} + \&c.$$

multiplying equation (1) by the similar one

$$e^{+^{\frac{1}{2}}\phi'} = \cos\phi' +^{\frac{1}{2}} \sin\phi'$$

we have

$$e^{+^{\frac{1}{2}}(\phi+\phi')} = \cos\phi \cos\phi' + \sin\phi \sin\phi' +^{\frac{1}{2}} [\cos\phi \sin\phi' + \sin\phi \cos\phi'].$$

But again, from (1) we have

$$e^{+^{\frac{1}{2}}(\phi+\phi')} = \cos(\phi + \phi') +^{\frac{1}{2}} \sin(\phi + \phi').$$

And since we are entitled to compare the *real* and *imaginary* parts in the two last equations, we conclude that

$$\cos(\phi + \phi') = \cos\phi \cos\phi' + \sin\phi \sin\phi'$$

$$\sin(\phi + \phi') = \cos\phi \sin\phi' + \sin\phi \cos\phi'.$$

These are the fundamental equations for the comparison of hyperbolic sines and cosines.

2. The distributive symbols $+^{\frac{1}{4}}$, $+^{\frac{1}{2}}$, $+^{\frac{3}{4}}$, $+$, enable us to construct the ordinary double algebra. The equation $+^{\frac{1}{2}}1 + 1 = 0$ makes $+^{\frac{1}{2}}$ *real* as well as $+$. But $+^{\frac{1}{4}}$ and $+^{\frac{3}{4}}$ are both *imaginary*, as compared with $+$ and $+^{\frac{1}{2}}$, though they admit of being compared together because of their relation

$$+^{\frac{1}{4}}1 +^{\frac{3}{4}}1 = 0,$$

which is a consequence of

$$+^{\frac{1}{2}}1 + 1 = 0.$$

3. With the distributive symbols $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, $+$, supposed to be *heterogeneous*, we might construct an algebra of one real and two imaginary symbols. This algebra would be virtually equivalent to the triple system discussed by Mr. Graves in former communications to the Academy.

4. Starting with the primary symbol $+^{\frac{1}{9}}$ we might frame an algebra with two imaginaries, viz., $+^{\frac{1}{9}}$ and $+^{\frac{2}{9}}$, and three reals, $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, $+$, related to one another by the condition

$$+^{\frac{1}{3}}1 +^{\frac{2}{3}}1 + 1 = 0.$$

Developing $e^{+^{\frac{1}{9}}\phi}$ we find it equal to

$$\lambda_\phi +^{\frac{1}{9}} \mu_\phi +^{\frac{2}{9}} \nu_\phi,$$

λ_ϕ, μ_ϕ, and ν_ϕ representing series in which the signs of the terms are successively $+$, $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, $+$, $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, &c. Between these series and the ordinary trigonometric series, expressing the sine and cosine in terms of the arc, there exist many remarkable analogies.

Mr. J. J. A. Worsaae, of Copenhagen, in continuation of his former communication to the Academy, gave a review of the different descriptions of Danish and Irish antiquities, and of